For large values of x, the Fresnel integrals may be approximated by:

$$\int_0^x \sin[k(\pi/2)y^2]\,dy \doteq 1/2(k)^{1/2} - (\pi kx)^{-1}\cos[k(\pi/2)x^2] \qquad (17)$$

$$\int_0^x \cos[k(\pi/2)y^2]\,dy \doteq 1/2(k)^{1/2} + (\pi kx)^{-1}\sin[k(\pi/2)x^2] \qquad (18)$$

If Eq. (13) is normalized by λ, integrated from x large to infinity, expanded into its real and imaginary components, it may be expressed as:

$$[\xi(x)-\xi(\infty)]/\lambda = (1/(2)^{1/2}\pi kx)\{\sin[k(\pi/2)x^2+\pi/4] - i\cos[k(\pi/2)x^2+\pi/4]\} \qquad (19)$$

The amplitude of the oscillation is recognized to be $1/(2)^{1/2}\pi kx$. This formula is only true for large values of x; however, Fig. 1 indicates very good agreement even for small values of x.

Concluding Remarks

This Note has presented a simple closed-form expression for determination of the final attitude dispersions of a symmetrical roll accelerated vehicle subjected to body fixed transverse torque. In addition, an expression describing the oscillatory envelope is also given. Comparison of the closed-form solutions with numerical integration of the equations indicates good agreement; hence, the closed-form solutions give the analyst a quick and accurate means of evaluating this problem.

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Suboptimal Controller for a Linearized n-Body Spacecraft

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Nomenclature

B	= diagonal damping matrix
$DEZ\{\ \}$	= vector deadzone function
E	= unit (identity) matrix
i	= integer used to designate a joint
k	= diagonal stiffness matrix
n	= number of rigid bodies in spacecraft model
r	= number of degrees of freedom
α	= matrix used in state equations [see Eq. (1)]
γ_k, γ	= relative angular motion at joint k; γ is $r-3\times 1$ vector having components γ_k, $k = 1, 2, \ldots, r-3$
θ_i, ω_{0_i}	= attitude angles of base body 0 (components of vector θ); angular velocity measure numbers of base body 0, $i = 1, 2, 3$ (components of vector ω_0)
$\omega_0, \omega_R, \omega$	= angular velocity vector of base body 0; relative angular velocity components $\dot\gamma_k$, $k = 1, 2, \ldots, r-3$; ω has components ω_0 and ω_R
$A_{11}, A_{12}, A_{21}, A_{22}$	= elements of partitioned matrix appearing in Eq. (1)
b_i	= basis vectors for base body 0, $i = 1, 2, 3$
B_i	= damping coefficients for joint i
$C_\theta, C_{\omega 0}, C_\gamma, C_{\omega R}$	= control gain matrices
$F_{\lambda_j}, M_{\lambda_j}$	= interaction force and moment on body λ due to joint j
F_λ, M_λ	= vectors representing externally applied forces and moments to body λ
K_i	= stiffness coefficients for joint i
L_0, L_R, L	= vector forcing function for base body; vector forcing function for $n-1$ remaining bodies; vector L has components L_0 and L_R
$\bar L_0, \bar L_R$	= vector forcing functions used in defining L; $\bar L_0$ and $\bar L_R$ are formed from L_0 and L_R by not including the terms $K\gamma + B\dot\gamma$
L_{00}, L_{0R}	= vector forcing functions used in defining $\bar L_0$; L_{00} is the contribution to $\bar L_0$ due to forces F_0 and moments M_0 applied to the base body; L_{0R} is the contribution made by forces F_λ and moments M_λ with $\lambda \neq 0$
L_{R0}, L_{RR}	= vector forcing functions used in defining $\bar L_R$; L_{R0} is the contribution to $\bar L_R$ due to F_0; L_{RR} is the contribution made by F_λ and M_λ with $\lambda \neq 0$
$\bar L_{00}$	= vector forcing function used in defining L_{00}. $\bar L_{00}$ is formed from L_{00} by not including the moment M_0
u_0, u_R	= suboptimal control vector for base body 0; suboptimal control vector for $n-1$ remaining bodies

Received August 2, 1973; revision received August 1, 1974. This work was prepared under NASA Contract NAS-7-100. Based on a paper presented at the AAS/AIAA Astrodynamics Conference, July 16–18, 1973, Vail, Colo.
Index category: Spacecraft Attitude Dynamics and Control.
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Introduction

FOR deep-space missions, the requirements placed on antenna pointing, articulation control, science platform settling times, etc., tend to become continually more stringent. Moreover, to meet the objectives of the scientific experiments and to provide isolation from the radiation produced by the power source, booms are frequently employed. These facts dictate that a suitable spacecraft model must be determined and, in addition, that sensor noise and plant disturbances be accounted for.

Considerable attention, in the open literature, has been focused on the problem of developing a suitable set of deterministic dynamical equations for a spacecraft.[1–6] Recently, a particularly elegant albeit complicated set of dynamical equations for an n-hinged rigid-body spacecraft has been developed.[2] The salient features of this set of dynamical equations are that 1) constraint torques do not appear, and 2) the number of variables involved is equal to the number of degrees of freedom of the system. Stochastic control theory has also been given special attention in the open literature. Reference 7 is devoted exclusively to linear stochastic optimal control of linear systems subject to the expected value of a quadratic cost function.

In the present work, the objective is to determine a controller which makes use of the elaborate, deterministic model of the spacecraft and, in addition, accounts for sensor noise, disturbances, etc. In essence, an optimal stochastic controller is sought. However, because of the practical importance of ease of implementation, simplicity, and reliability, a suboptimal stochastic controller is determined.

It is known that a solution can be found to a linear stochastic optimization problem involving a quadratic cost functional. However, the plant representing the dynamics of the spacecraft is nonlinear. In addition, a single quadratic cost function which accounts for *all* of the desired characteristics of the controller *cannot* be found. Nevertheless, a suboptimal stochastic controller is obtained by: 1) appropriately linearizing the dynamical model of the spacecraft to obtain the plant; 2) invoking the "certainty-equivalence" principle of modern control theory to arrive at the structure of the optimal linear controller; 3) generating the suboptimal controller.

The contributions of this paper include the casting of the dynamical equations for the spacecraft in a form suitable for optimal stochastic control theory and the development of a suboptimal stochastic controller. To the writer's knowledge, a stochastic controller based on a realistic model of a complex spacecraft has not been previously obtained.

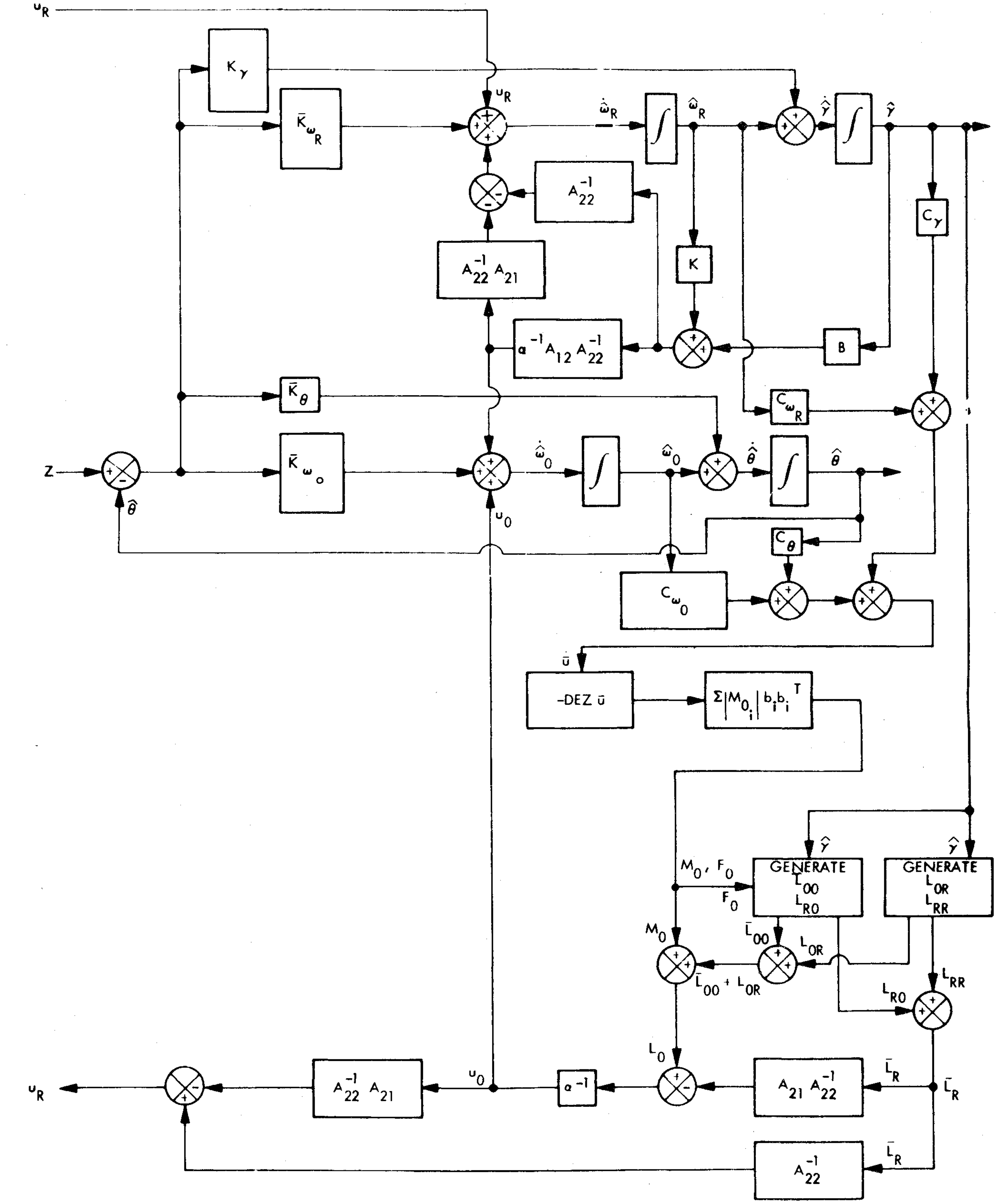

Fig. 1 Block diagram of suboptimal controller for a linearized n-body spacecraft.

Linearized Dynamical Equations for an N-Hinged Rigid-Body Spacecraft

In this section, the linearized dynamical equations for an n-hinged rigid-body spacecraft are provided. In Refs. 1–3, the dynamical equations for an n-body vehicle are developed. Starting with these equations, a linearized set of state equations can be developed (see Refs. 5 and 6). In vector-matrix notation, the state equations become†

† See Definitions of Symbols for definitions of all symbols used in this paper.

$$\begin{pmatrix} \dot{\theta} \\ \dot{\omega}_0 \\ \dot{\gamma} \\ \dot{\omega}_R \end{pmatrix} = \begin{bmatrix} 0 & E & 0 & 0 \\ 0 & 0 & \alpha^{-1}A_{12}A_{22}{}^{-1}K & \alpha^{-1}A_{12}A_{22}{}^{-1}B \\ 0 & 0 & 0 & E \\ 0 & 0 & -A_{22}{}^{-1}(E+A_{21}\alpha^{-1}A_{12}A_{22}{}^{-1})K & -A_{22}{}^{-1}(E+A_{21}\alpha^{-1}A_{12}A_{22}{}^{-1})B \end{bmatrix} \begin{pmatrix} \theta \\ \omega_0 \\ \gamma \\ \omega_R \end{pmatrix} + \begin{bmatrix} 0 \\ \alpha^{-1}\bar{L}_0 - \alpha^{-1}A_{12}A_{22}{}^{-1}\bar{L}_R \\ 0 \\ A_{22}{}^{-1}(E+A_{21}\alpha^{-1}A_{12}A_{22}{}^{-1})\bar{L}_R - A_{22}{}^{-1}A_{21}\alpha^{-1}\bar{L}_0 \end{bmatrix} \quad (1)$$

where

$$\alpha = [A_{11} - A_{12}A_{22}{}^{-1}A_{21}] \quad (2)$$

Stochastic Controller for Multihinged Rigid-Body Spacecraft

In this section, a stochastic controller based on the dynamical model presented above is given. The form of the stochastic controller appropriate for a linear problem subject to a quadratic cost functional is retained; however, the form of the control function for this special case is passed through desirable nonlinearities peculiar to attitude control before being applied to the plant (see Fig. 1).

In this analysis, the suboptimal vector function u_0 for the base body is obtained by

1) Generating the actuating signal $\bar{u}$ used to fire the thrusters located on the base body according to (note that $\bar{u}$ retains the structure of the optimal u^* for a linear problem but the control gains are obviously not based on the linear formulation)

$$\bar{u} = C_\theta \hat{\theta} + C_{\omega_0}\hat{\omega}_0 + C_\gamma \hat{\gamma} + C_{\omega_R}\hat{\omega}_R \quad (3)$$

2) Passing the function $\bar{u}$ through a vector deadzone function to obtain the applied moment M_0 and the associated applied thrust F_0 acting on the base body; this contribution to u_0 is given by

$$M_0 = -\left[\sum_{i=1}^{3} |M_{0_i}| \, b_i b_i{}^T\right] DEZ\,\bar{u} \quad (4)$$

3) Generating the terms $\bar{L}_{00}$, L_{0R}, and L_{R0}. The vector control function u_0 is given by

$$u_0 = \alpha^{-1}\left[-\sum_{i=1}^{3} |M_{0_i}| \, b_i b_i{}^T\right] DEZ\,\bar{u} + \alpha^{-1}\bar{L}_{00} + \alpha^{-1}L_{0R} - \alpha^{-1}A_{12}A_{22}{}^{-1}(L_{R0} + L_{RR}) \quad (5)$$

Correspondingly, the vector control function for the remaining $n-1$ bodies is given by

$$u_R = -A_{22}{}^{-1}A_{21}u_0 + A_{22}{}^{-1}(L_{R0} + L_{RR}) \quad (6)$$

Note that the time-varying functions $\bar{L}_{00}$, L_{0R}, L_{R0}, and L_{RR} are based on $\hat{\gamma}$. The control gains C_θ, C_{ω_0}, C_γ, C_{ω_R} are, of course, selected so that the pointing requirements are met.

Description and Uses of Suboptimal Stochastic Controller

From Fig. 1 it is seen that the stochastic controller consists of a Kalman filter adjoined to the generators of u_0 and u_R. The measurement vector z, in this paper, consists of the sensed attitude angles of the base body. The estimated state $\hat{x}$ consisting of $\hat{\theta}$, $\hat{\omega}_0$, $\hat{\gamma}$, $\hat{\omega}_R$ is used to generate the actuating signal $\bar{u}$ which fires the thrusters located on the base body. In essence, the applied moment M_0 tends to null a weighted combination of the attitude and angular velocity of the base body and the relative motion of the remaining $n-1$ bodies. Note that, if it is not desirable to null a specific relative motion γ_k and its associated rate $\dot{\gamma}_k$, then the appropriate components of the control gain matrices C_γ and C_{ω_R} are zero. Note, too, that the system matrices A_{11}, A_{22}, A_{12}, A_{21}, α are constant. The Kalman gain matrices $\bar{K}_\theta$, $\bar{K}_{\omega_0}$, $\bar{K}_{\omega_R}$, $\bar{K}_\gamma$ and the control gain matrices C_θ, C_{ω_0}, C_{ω_R}, C_γ can be approximated by piecewise constant functions, if it is so desired.

This stochastic controller will be used to study the effects of interactions of an articulated science platform (undergoing small-angle slews) on the base body motion, and to study the effects of interactions of booms (undergoing small oscillations) on thrust vector control performance. Specifically, this controller will be used to analyze the Mariner Jupiter Saturn (MJS '77) spacecraft in the cruise, the thrust vector control, and the articulation control modes. The MJS '77 stringent accuracy requirements and settling times associated with the articulated science platform dictates that an elaborate dynamical model be used and that disturbances and sensor noise be properly accounted for. It is planned that the computational aspects and simulation results obtained from this formulation will be published in the near future.

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Optimal Payload Ascent Trajectories of Winged Vehicles

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Nomenclature

$\bar{m}$ = mass at burnout normalized by W

$\overline{q\alpha}$ = product of dynamic pressure and angle of attack normalized by the reference condition value of 33 516 N-deg/m^2 (700 lb-deg/ft^2) (see Fig. 2)

r = recovery resizing factor, vehicle inert mass growth per unit mass of subsystem growth

W = wing mass at the reference condition, 5149 kg (11,352 lb)

$\Delta\bar{W}$ = change in wing mass normalized by W

Received May 1, 1974; revision received August 13, 1974.

Index categories: Launch Vehicle and Missile Trajectories; Launch Vehicle and Missile Structural Design.

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